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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

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11[70F25, 34A09, 65L80]—*Nonholonomic motion of rigid mechanical systems from a DAE viewpoint*, by Patrick J. Rabier and Werner C. Rheinboldt, SIAM, Philadelphia, PA, 2000, viii+140 pp., 25 cm, softcover, $36.00

Over centuries people have been fascinated by the problem of how to determine the motion of systems of rigid bodies or mass points that are subject to external forces and constraints. Today the computer aided systematic generation of the equations of motion and specially adapted numerical solution techniques form the backbone of modern multibody system simulation tools that are successfully used in vehicle dynamics, robotics, and biomechanics.

In the book under review the most general case of nonconservative three-dimensional multibody systems with mixed holonomic and nonholonomic constraints is considered in detail. Generalizing the Gauss principle of least constraint, the equations of motion may be derived as second-order differential equations that are supplemented by nonlinear constraints (differential-algebraic equations, DAEs). Based on ideas of modern DAE theory, these equations are studied analytically and a new approach to the construction of time integration methods is proposed.

The book has nine chapters and starts with an Introduction and short review of the state-of-the-art in Chapter 1. The theoretical results are developed step by step in Chapters 2–7. Numerical solution methods and computational examples are the topics of Chapters 8 and 9.

In Chapter 2 principles of classical mechanics and their application to constrained systems of mass points are discussed providing, at a rather elementary level, background for the substantially more complex analysis of rigid bodies. In Chapter 3 the confirmation space $\mathcal{C}_0$ of a rigid body is studied to include the rotational degrees of freedom. From the numerical point of view, the use of quaternions, i.e., $\mathcal{C}_0 = \mathbb{R}^3 \times S^3 \subset \mathbb{R}^3 \times \mathbb{R}^4$ is found to be favourable. In Chapter 4 this representation of $\mathcal{C}_0$ is applied to show that the equations of motion for an unconstrained rigid body may be written as a second-order ordinary differential equation on $\mathbb{R}^3 \times S^3$. This is the essential prerequisite for the analysis of constrained systems of rigid bodies in Chapter 5 since now the generalized Gauss principle may be used to derive the equations of motion in DAE form.

Because of $\mathbb{R}^3 \times S^3 \subset \mathbb{R}^3 \times \mathbb{R}^4$, these equations may be considered both as a classical DAE in $\mathbb{R}^n$ and as a DAE on a manifold $\mathcal{M} \subset \mathbb{R}^n$. Both approaches are considered separately in Chapters 6 and 7 to prove for initial value problems the existence and uniqueness of a solution. This analysis is based on a formulation $\Gamma(t, x, \dot{x}) = 0$ of the constraints that involves position coordinates x as well as velocities $\dot{x}$ (the *index*-2 *formulation* in DAE terminology [2]). It is assumed that the Jacobian $D_{\dot{x}}\Gamma$ has full rank. Therefore, holonomic constraints or more generally all geometric constraints $\Gamma(t,x) = 0$ do not fit a priori into this framework since $D_{\dot{x}}\Gamma = 0$. For these systems it is proposed to substitute $\Gamma(t,x(t)) = 0$ by its time derivative (*index reduction* in classical DAE theory). In Sections 6.2 and 7.3 this strategy is carefully extended to general systems with mixed holonomic and nonholonomic constraints.

As soon as the equations of motion are given in DAE form they may be solved by DAE methods. An approach that is based on local parametrizations of manifolds is discussed in more detail in Chapter 8. The successful practical application of this method is nicely illustrated by numerical tests for typical nonholonomic examples from the literature (Chapter 9). Finally, an appendix was added to make the book essentially self-contained. This appendix summarizes shortly some material on submanifolds of finite-dimensional spaces.

In less than 150 pages this book provides a compact discussion of a very general class of mechanical problems and gives a strong theoretical justification for a DAE formulation of the equations of motion. It is very helpful that the clear and detailed mathematical presentation refers frequently to simple special cases, like systems of mass points, single unconstrained rigid bodies or planar systems to explain the ideas and technical problems of this analysis.

The topic of this book is not restricted to the mathematical background of a classical mechanical problem. The extension of classical DAE theory to DAEs on manifolds in Chapter 7 has independent value since this analysis covers a much larger class of constrained problems.

Readers who consider the terms holonomic and nonholonomic constraint in the classical sense of H. Hertz as antonyms will be surprised to find a complete analysis of the holonomic case and the mixed holonomic/nonholonomic case in a book entitled "Nonholonomic motion...". However, in one of the first paragraphs of the Introduction, the use of the terms holonomic, nonholonomic, geometric, and kinematic constraints is clarified.

The publisher claims that "mechanical engineers and robotics engineers will find this book valuable", but the potential reader should be aware that this is a book written by mathematicians in a way that is typical of *mathematical* presentations. References to the work of E. J. Haug [3] create a link to computational mechanics.

Besides the theoretical results, the book contributes also to two central practical problems in multibody dynamics: the choice of coordinates and the efficient numerical solution of the equations of motion. There is a vast literature on both topics and several different strategies have been developed and implemented successfully over the last two decades ("... nothing can really be new that addresses the motion of rigid bodies ..., page vii). In view of the state-of-the-art, it is questionable to present one specific choice of coordinates (quaternions) as the "correct" one (pages vii, 2 and Chapter 3).

Furthermore, the efficiency of the new numerical methods of Chapter 8 should have been compared with standard DAE methods for multibody systems like BDF [1, Section 6.2] or implicit Runge-Kutta methods [2, Chapter VII]. The initial statement of Section 8.3, "Standard DAE software, such as the widely used code DASSL ... are certainly not useable for the production solution of the DAEs (6.1)", is wrong. DASSL has been used very successfully in academic research and industrial multibody software for more than a decade (see, e.g., [4]).

Despite these critical remarks there is no doubt that the book will help to decrease the gap between abstract differential geometry and its applications in computational mechanics. It may be recommended to all mathematicians and engineers who are interested in the theoretical analysis of constrained mechanical systems and in practical applications of differential geometry.

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12[90C30, 90C25, 65K05]—*Trust-region methods*, by Andrew R. Conn, Nicholas I. M. Gould, and Philippe L. Toint, SIAM, Philadelphia, PA, 2000, xix+959 pp., 26 cm, softcover, $119.00

This giant monograph is the first book (until now, it is also the only one) published on trust-region methods. Trust-region methods are a class of numerical methods for solving nonlinear optimization problems. These methods are reliable and robust, they can be applied to ill-conditioned problems, and they have very strong convergence properties. The authors are three distinguished researchers having long been involved in the development and implementation of algorithms for large-scale numerical optimization. They were corecipients of the 1994 Beale–Orchard–Hays prize for their work on the LANCELOT optimization package.

The aims of the book are best stated by the authors in the Preface:

> *Three major aims are, firstly, a detailed description of the basic theory of trust-region methods and the resulting algorithms for unconstrained, linearly constrained, and generally constrained optimization; secondly, the inclusion of implementation and computational details; and finally, substantive material on less well-known advanced topics, including structured trust regions, derivative-free methods, approximate methods (including noise), nonmonotone algorithms, and nonsmooth problems.*

Chapter 1 is a brief introduction, which gives a description of fundamental trust-region ideas, overviews the history of trust-region methods, and tables some references of applications of trust-region methods in science and engineering. Chapters 2 to 5 are some background mathematics, including vector spaces, matrix analysis, optimality conditions, and methods for solving linear systems and eigenproblems.